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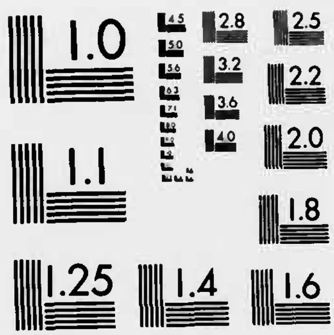

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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20. Abstract (continued)

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The effect of the relative position and length of the crack, with respect to the plate width, on the eddy current density near the tips of the crack is given special attention. These results may be useful to simulate eddy current flow detection phenomena.

A BOUNDARY INTEGRAL METHOD FOR EDDY CURRENT FLOW AROUND CRACKS IN THIN PLATES

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ABSTRACT

A boundary element method which employs a Green's function for a crack has been developed to calculate the induced eddy current flow around cracks in thin conducting plates. The theoretical equations employ a stream function for the current density vector and is equivalent to the electric field vector potential method. A low frequency or large skin depth approximation leads to a Poisson equation for steady harmonic inductor fields. Induced currents around a crack in a square plate due to a uniform inductor field for various crack positions and sites have been calculated in this paper.

The effect of the relative position and length of the crack, with respect to the plate width, on the eddy current density near the tips of the crack is given special attention. These results may be useful to simulate eddy current flow detection phenomena.

INTRODUCTION

The boundary element method (BEM) (also called the boundary integral equation method) has emerged as an important computational technique for electrodynamic problems. Wu et al [1] and Ancelle et al [2] have addressed magnetostatic problems by the BEM while Trowbridge [3] has considered problems by the magnetic potential method. Very recently, Salon and Schneider [4] have solved problems of eddy current flow in long prismatic conductors by the BEM based on the electric potential approach.

In this paper, we describe a powerful boundary element technique for calculating induced eddy current flows in conducting plates with through cracks using the electric potential approach. The BEM has the important advantage that only the boundary of a body (rather than the entire domain) needs to be discretised in a numerical solution procedure.

There have been some attempts to model eddy current flow around annular cracks in rods and in plates by replacing cracks by slots (see for example Ref. [5]). However, we have shown that the induced current in the vicinity of a crack leads to a singularity of current density at the crack tips [6,7]. This high concentration allows one to use eddy current testing devices such as active and passive search coils to detect the presence of cracks. It also results in a temperature hot spot which can be detected by infrared scanning [6,8]. The boundary element technique introduced by the authors [6,7] and described here allows one to model exactly the singular nature of current density at crack tips of thin plates. This technique can handle any arbitrary shape of the plate and general magnetic fields.

In this paper we discuss application of the BEM to eddy current flow in a cracked square plate due to an uniform inductor field applied normal to the plate. A number of crack sizes to plate size configurations has been considered. Also, effect of the relative position of a crack tip to the plate edge on the induced eddy current distribution has been investigated.

GOVERNING EQUATIONS

A thin plate with a crack in it is shown in Fig. 1. The plate is made of a conducting material of conductivity σ. The plate boundary can be arbitrary and its thickness (uniform) is h. The thin line crack is of length $2a$ and can have arbitrary orientation relative to the boundary of the plate. The coordinate system for the problem is also shown in Fig. 1. The origin of coordinates lies at the center of the crack and at the midsurface of the plate.

An external, oscillatory magnetic field, $\underset{\sim}{B}^{o}$, is applied which induces a current density $\underset{\sim}{J}$ in the plate. It is assumed that the current density is uniform across the plate thickness and that the skin depth (which is inversely proportional to the square root of the frequency) is large compared to the plate thickness.

A stream function (or electric potential) formulation is used in this problem. The stream function, $\psi(x_1,x_2)$, is defined as

$$\underline{J} = \nabla\times(\psi\underline{k}) = -\underline{k}\times\nabla\psi \quad (1)$$

This equation guarantees the conservation of charge equation $\nabla\cdot\underline{J} = 0$ for charge free regions.

Using Ohm's law the governing differential equation for the stream function is obtained as [6,7]

$$\nabla^2\psi = \sigma\frac{\partial}{\partial t}(B_3^o+B_3^1) \quad (2)$$

In the above, B_3^1 is the self magnetic field due to the current $\underline{J}$. It has been shown in ref. [9], however, that for a sinusoidal applied field, with the skin depth much greater than the thickness of the plate, B_3^1 can be neglected relative to the applied field B^o. This assumption simplifies the problem, and, with $B_3^o = \hat{B}_3^o e^{i\omega t}$ (with $i = \sqrt{-1}$ and ω the frequency), the spatial part of ψ satisfies a two-dimensional nonhomogeneous Poisson's equation

$$\nabla^2\psi = i\omega\sigma\hat{B}_3^o = f(x_1,x_2) \quad (3)$$

The boundary condition requires that the current must be tangential to the plate boundary. Thus ψ is required to be constant on the boundaries ∂C_1 and ∂C_2. On one boundary, the value of ψ is set to zero, while on the other boundary $\psi = C$ and C is obtained from the assumption that the net flux flowing through the crack boundary is zero. This leads to the condition

$$\oint_{\partial C_1} \underline{J}\cdot\underline{t}\,ds = 0 \quad (4)$$

where $\underline{t}$ is an unit tangent to ∂C_1 and s is the distance measured along a boundary in the anticlockwise sense. This formulation assumes that no current flows across the crack or crack tip and leads to a singularity of the $\underline{J}$ field at a crack tip. This is analogous to the stress singularity in fracture mechanics. It is possible that some leakage of current occurs across a crack tip and thus relieves the singularity in actual conductors. Possible leakage of current is not considered in this paper. (It is noted here that infrared scans of eddy current flow around cracks do indeed show a large increase in temperature at the crack tips, indicating high current density at the crack tips [6].)

In summary, the boundary conditions on ψ, used in this formulation, are

$$\psi = 0 \text{ on the crack boundary } \partial C_1 \quad (5)$$

$$\frac{d\psi}{ds} = 0 \text{ on the outside boundary } \partial C_2 \quad (6)$$

$$\oint_{\partial C_1} \frac{d\psi}{dn}\,ds = 0 \quad (7)$$

These boundary conditions, together with the field equation (3), constitute a well posed problem.

BOUNDARY ELEMENT FORMULATION

Integral equations

An integral equation formulation for Poisson's equation (3) can be written as (Fig. 1) [6,7]

$$2\pi\psi(p) = \oint_{\partial C_2} K(p,Q)G(Q)ds_Q + \int_A K(p,q)f(q)dA_q \quad (8)$$

This is a single layer potential formulation where G, a source strength function on the outside boundary, must be determined from the boundary condition on it (equation 9). The points p (or P) and q (or Q) are source and field points, respectively, with capital letters denoting points on the boundary of the body and lower case letters denoting points inside the body. The area of the body B is denoted by A.

It has been shown [6] that ψ from equation (8) with the following kernel satisfies the boundary conditions (5) and (7) implicitly.

$$K(p,q) = \mathrm{Re}[\phi(z,\bar{z},z_o)] \quad (9)$$

$$\phi(z,\bar{z},z_o) = \ln(1-r_1/\xi) - \ln(1-r_1\bar{\xi}) \quad (10)$$

where $r_1 = \dfrac{z_o \pm \sqrt{z_o^2-4}}{2}$, $|r_1| \le 1$

$$\xi = \frac{z \pm \sqrt{z^2-4}}{2}, \quad |\xi| \le 1$$

Re denotes the real part of the complex argument, z and z_o are the source and field point coordinates, respectively, in complex notation and a superposed bar denotes, as usual, the complex conjugate of a complex quantity.

The remaining boundary condition (6) on the outside surface is satisfied by using a differentiated version of (8) and taking the limit as p inside B approaches a point P on ∂C_2. Defining

$$H_1 = \mathrm{Im}\left(\frac{\partial\phi}{\partial z} - \frac{\partial\phi}{\partial\bar{z}}\right), \quad H_2 = -\mathrm{Re}\left(\frac{\partial\phi}{\partial z} + \frac{\partial\phi}{\partial\bar{z}}\right) \quad (11)$$

the boundary condition (6) becomes

$$0 = \oint_{\partial C_2} H_i(P,Q)n_i(P)G(Q)ds_Q + \int_A H_i(P,q)n_i(P)f(q)dA_q \quad (12)$$

where n_i are the components of the unit outward normal to ∂C_2 at some locally smooth point on it.

The current, $\underline{J}$, at a point inside the body is obtained from equations (1) and (8).

Discretization of equations and solution strategy

The outer boundary of the body, ∂C_2, is divided into N_2 straight boundary elements using N_b ($N_b = N_2$) boundary nodes and the interior of the body, A, is divided into n_1 triangular internal elements. A discretized version of equation (12) is

$$0 = \sum_{N_2} \int_{\Delta s_i} H_i(P_M,Q) n_i(P_M) G(Q) ds_Q + \sum_{n_1} \int_{\Delta A_i} H_i(P_M,q) n_i(P_M) f(q) dA_q \qquad (13)$$

where P_M is the point P where it coincides with a node M at a center of a boundary segment on ∂C_2 and Δs_i and ΔA_i are boundary and internal elements respectively.

A simple numerical scheme is used in which the source strengths G are assumed to be piecewise uniform on each boundary segment with their values to be determined at the nodes which lie at the centers of each segment. Substitution of the piecewise uniform source strengths into equation (13) and carrying out of the necessary integrations, analytically and numerically, leads to an algebraic system of the type

$$\{0\} = [A]\{G\} + \{d\} \qquad (14)$$

The coefficients of the matrix [A] contain boundary integrals of the kernel. The vector {d} contains contributions from the area integrals and the vector {G} the unknown source strengths at the boundary nodes. The dimension of {G} depends only on the number of boundary elements on ∂C_2 and the internal discretization is necessary only for the evaluation of integrals with known integrands.

The solution strategy is as follows. The matrix [A] and vector {d} in equation (14) are first evaluated by using the appropriate expressions for the kernels and the prescribed function f in equation (3). Equation (14) is solved for the vector {G}. This value of {G} is now used in a discretized version of equation (8) to obtain the values of the stream function ψ at any point p. Finally, the current vector at any point is obtained from equations analogous to (8).

NUMERICAL RESULTS

In the numerical computations, $\hat{B}_3^o$ in Eq. (13) is assumed to be a constant. Eq. (3) can be non-dimensionalized to the form

$$\nabla^2\hat{\psi}(\hat{x}_1,\hat{x}_2) = 1 \quad , \quad \hat{x}_i = x_i/a \qquad (15)$$

where

$$\hat{\psi} = \frac{\psi\mu_o}{14\pi B_3^o R} \qquad R = \frac{2a^2}{\pi\delta^2} \text{ and the skin depth}$$

$$\delta = \sqrt{\frac{2}{\omega\sigma\mu_o}} \qquad \hat{J} = \frac{Ja\mu_o}{12\pi B_3^o R}$$

For the results in this paper a = 2. A typical mesh for the results for example shown in Fig. 2d has 48 boundary segments uniformly distributed along the upper half (due to symmetry) of the boundary of the plate. In order to evaluate the known area integral in Equation 13, the internal area quadrature was used. It took about 300 c.p.u. secs on IBM 370/168 to obtain the results in Fig. 2d.

The equation (15) is identical to one relating to the torsion of shafts. The BEM was verified by comparing the numerical results for the solution of (15) in a square plate without a crack to known analytical results for the torsion of a shaft. The BEM method has also been checked against a finite element technique developed for eddy current problems [10].

Eddy current stream lines ($\hat{\psi}$ lines) are shown in Figs. 2 and 3 for a square plate with a crack in it. Fig. 2 (a) - (c) shows how the stream lines are affected by varying the size of the plate while keeping the crack size same. Due to symmetry only the upper half of the plate is shown in Fig. 2. Fig. 2 (d) shows the effect of moving the crack towards one of the plate edges. Fig. 3 shows a close up of the stream lines near right crack tip for Fig. 2 (c). The crowding of stream lines near crack tips leads to large gradient of $\hat{\psi}$ and therefore large induced currents in this region. The local temperature is proportional to the square of the current density ($\hat{J}\cdot\hat{J}$). Figure 4 shows calculated temperature scans along a line slightly above the crack ($\hat{x}_2 = .0125$) for the results shown in Fig. 2. From Figs. 4 (a) - (c) one can conclude that as the crack size increases relative to the plate size the hot spots at crack tips are more significant compared to those at the edges. The effect of moving the crack near the plate edge gives rise to significant hot spots as shown in Fig. 4 (d) and (c). This becomes more apparent when we look at the 'Eddy Current Intensity Factor' defined below. It has been shown [6,7] that the eddy current density squared is inversely proportional to the distance r from a crack tip. We can define an eddy current intensity factor, M_{III} as

$$\hat{J}^2 = M_{III}\frac{a}{r}$$

Table 1 shows the calculated values of M_{III} for the two crack tips for the results shown in Fig. 2. It is seen that the value of M_{III} remains practically constant for varying plate sizes. However it changes significantly as a crack tip is brought near an edge of the plate.

Table 1. Stress Intensity Factor MIII

$\frac{a}{L}$	$\frac{2x_1^o}{L}$	Right Crack Tip	Left Crack Tip	Figures 2, 4
0.05	1.0	0.125	0.125	(a)
0.10	1.0	0.130	0.130	(b)
0.25	1.0	0.145	0.145	(c)
0.10	0.6	3.96	1.30	(d)
0.10	0.3	15.45	6.93	(e)

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6. Mukherjee, S., Morjaria, M. and Moon, F.C., 'Eddy Current Flows Around Cracks in Thin Plates for Nondestructive Testing', accepted for publication in the ASME Journal of Applied Mechanics.

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9. Yuan, K.Y., Moon, F.C. and Abel, J.F., "Numerical Solutions for Coupled Magnetomechanics", Department of Structural Engineering and Theoretical and Applied Mechanics, Cornell University, February 1, 1980. See also, "Magnetic Forces in Plates Using Finite Elements", Proceedings of the Third Engineering Mechanics Division Specialty Conference, ASCE, Austin, Texas, September 1979, pp. 730-733.

10. Yuan, K.Y., Abel, J.F. and Moon, F.C., 'Eddy current calculations in thin conducting plates using a finite element stream function code', COMPUMAG, Sept. 1981.

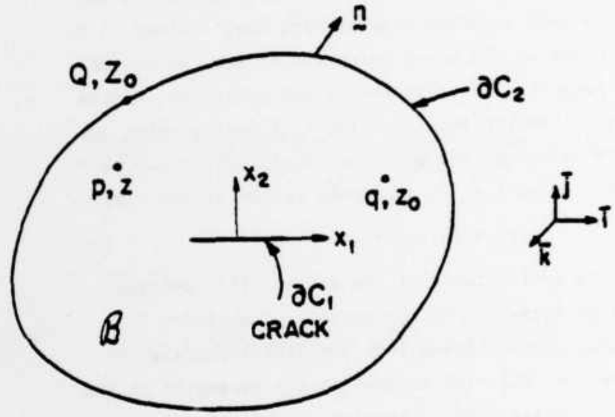

Figure 1. Cracked Plate.

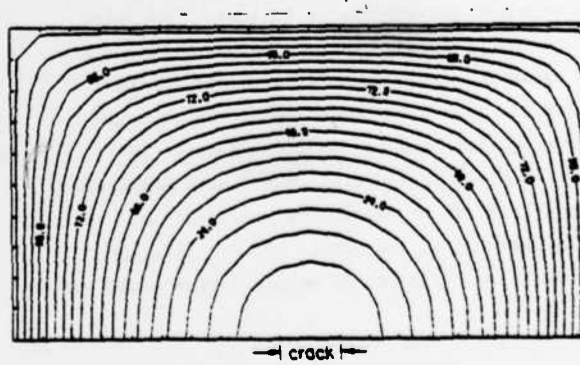

Fig. 2 (a).

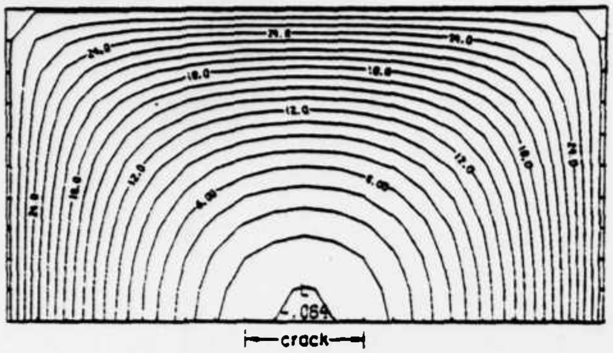

Fig. 2 (b).

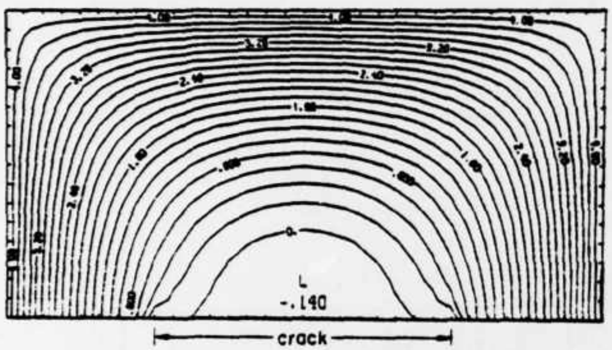

Fig. 2 (c).

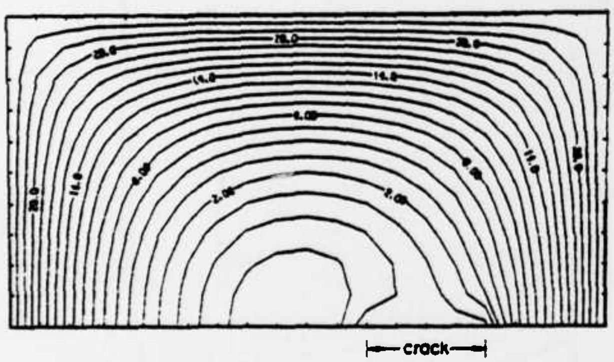

Fig. 2 (d).

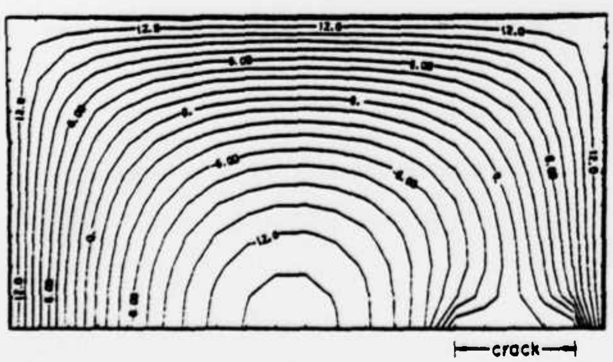

Fig. 2 (e).

Fig. 2. Eddy current stream lines in a square plate with a crack induced by an uniform magnetic field.

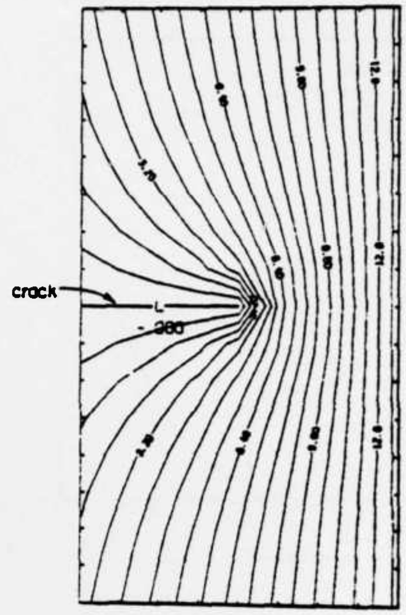

Fig. 3. Close up of Fig. 2 (e).

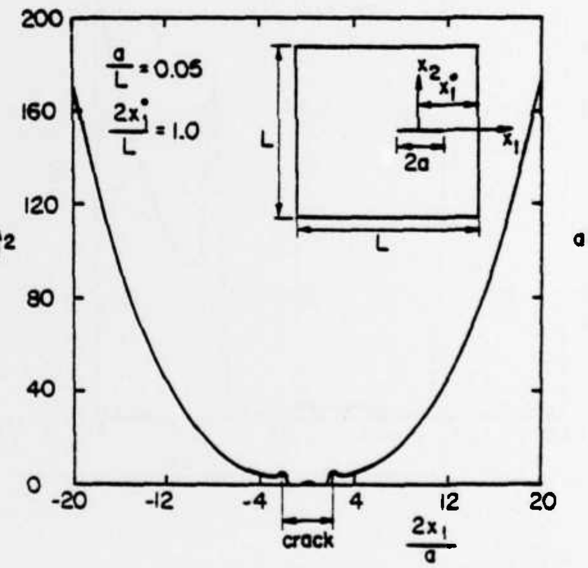

Fig. 4 (a).

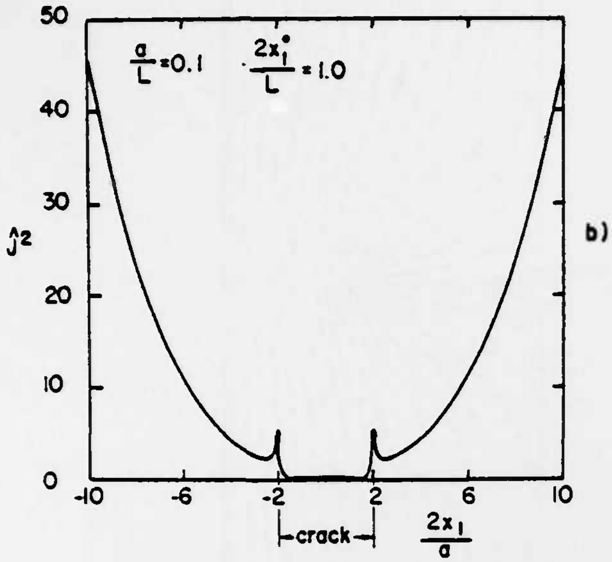

Fig. 4 (b).

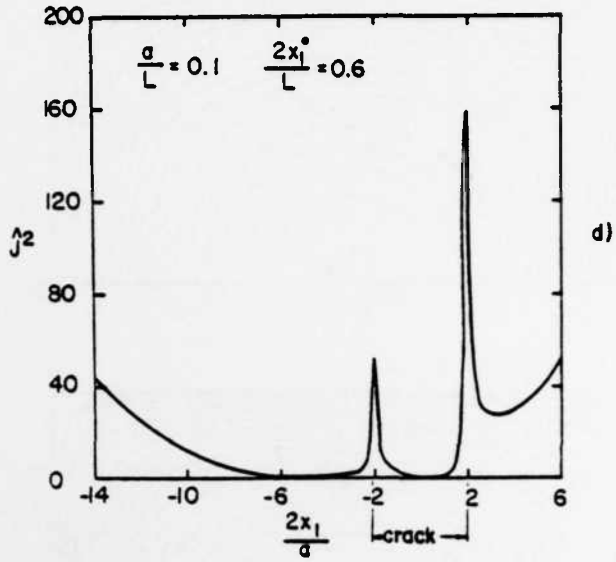

Fig. 4 (d).

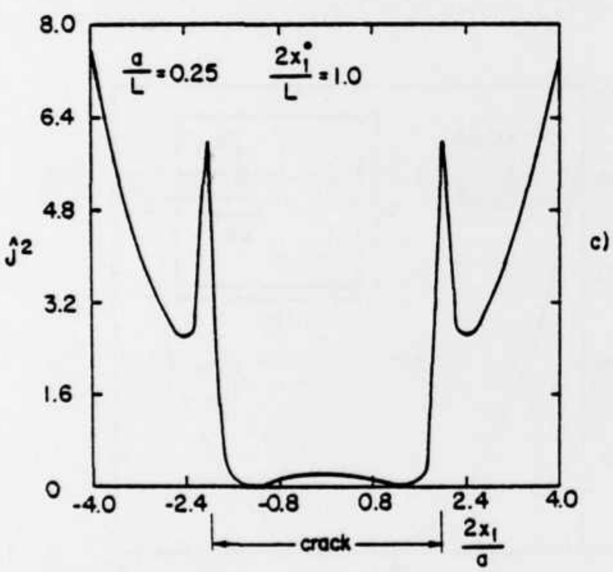

Fig. 4 (c).

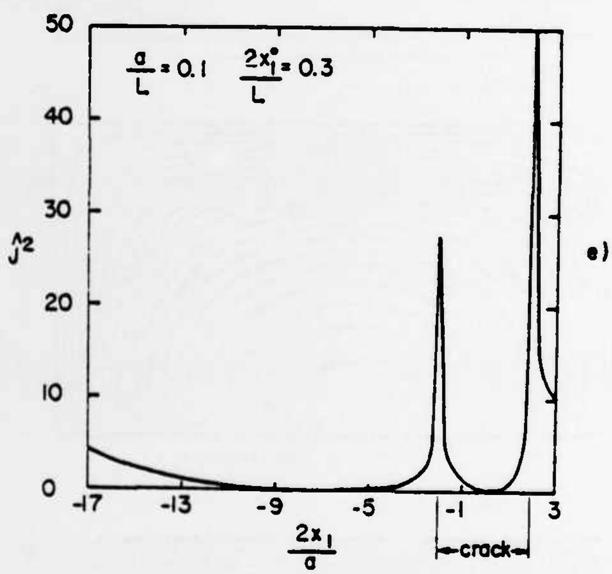

Fig. 4 (e).

Fig. 4. Joule heating intensity ($\hat{J}^2$) on sections $\frac{x_2}{a} = 0.0125$ shown in Fig. 2.

COMPOSITE LIST OF TECHNICAL REPORTS

TO THE

OFFICE OF NAVAL RESEARCH

NUMERICAL SOLUTIONS FOR COUPLED MAGNETOTHERMOMECHANICS

Task Number NR 064-621

Departments of Structural Engineering and
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1. K.Y. Yuan, F.C. Moon, and J.F. Abel, "Numerical Solutions for Coupled Magnetomechanics", Department of Structural Engineering Report Number 80-5, February 1980.

2. F.C. Moon and K. Hara, "Detection of Vibrations in Metallic Structures Using Small Passive Magnetic Fields", January 1981.

3. S. Mukherjee, M.A. Morjaria, and F.C. Moon, "Eddy Current Flows Around Cracks in Thin Plates for Nondestructive Testing", March 1981.

4. K.Y. Yuan, F.C. Moon, and J.F. Abel, "Finite Element Analysis of Coupled Magnetomechanical Problems of Conducting Plates", Department of Structural Engineering Report Number 81-10, May 1981.

5. F.C. Moon, "The Virial Theorem and Scaling Laws for Superconducting Magnet Systems", May 1981.

6. K.Y. Yuan, "Finite Element Analysis of Magnetoelastic Plate Problems", Department of Structural Engineering Report Number 81-14, August 1981.

7. K.Y. Yuan et al., "Two Papers on Eddy Current Calculations in Thin Plates", September 1981.

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